Generation of Generalized Meshes by Extrusion from Surface Meshes of Arbitrary Topology

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Abstract

A novel algorithm to extrude smooth, near-body volume meshes from surface meshes of arbitrary topology is presented. These meshes are classified as generalized meshes because multiple element topologies may be present within the same mesh. The algorithm utilizes a three-step, parabolic scheme based on the Poisson equation used in structured grid generation to extrude the volume mesh. Several preliminary example meshes are included to demonstrate the efficacy of the approach.

Introduction

Mesh generation operates fundamentally by distributing points throughout the volume of a physical region, as well as on its bounding surfaces. Connection of the points forms the mesh and subdivides the physical region into a filling set of discrete volume elements. Structured grids [1]-[5] and unstructured meshes [6]-[8] have been used successfully to solve a wide range of problems in computational field simulation. Each of these mesh types has advantages and disadvantages that have been well documented. It seems apparent that no single mesh type can simultaneously address the conflicting requirements of solution accuracy, computational efficiency, and automation of the grid generation process.

An alternative to uniform-topology meshing is the so-called hybrid grid method [9]-[12]. Elements of hybrid grids are not required to be of the same topological type. Typically, hybrid grid methods employ structured quadrilateral- or unstructured, triangular-element meshes to discretize interior bounding surfaces.

These surface meshes are then extruded into the domain forming a region of hexahedral or prismatic cells, respectively. Tetrahedra are used to fill the remaining voids in the domain. In the case of hexahedral near-body elements, a pyramidal transition layer is required to provide the triangular faces necessary for the tetrahedral mesh generation. Hybrid grids are attractive because of their flexibility with respect to automation as well as feature resolution through the use of anisotropic elements. Additionally, hybrid topologies require significantly fewer elements than unstructured meshes to achieve the same degree of resolution [9].

However, most hybrid grid generation technologies require that the surface discretization be uniformly of the same topological type. If an isotropic triangulation is used, an excessively large number of faces may be needed to adequately discretize the surface thereby affecting solution efficiency. If structured quadrilaterals are used, a manual surface decomposition must be performed that may require significant effort.

In this paper, we present an approach combining elements of both structured grid and unstructured mesh generation that we categorize as generalized mesh generation [13]-[15]. The generalized mesh topology places no restriction on the number of edges used to define a face or the number of faces used to define a cell. The advantage of using this topology is that mesh elements may be used that are topologically appropriate for the region being discretized. In this context, we define topologically appropriate as simultaneously meeting the requirements of solution accuracy, computational efficiency, and automation of the grid generation process. As with hybrid grids, a near-body mesh is generated by extrusion followed by a tetrahedral mesh generation to fill voids remaining in the domain.

Here we focus on the generation of the near-body mesh using the extrusion technique described in [14] and [15] modified to permit the surface meshes to be of arbitrary topology. The algorithm utilizes a parabolic marching scheme [3]-[5] to extrude the mesh in layers starting from an initial surface mesh. An approach of this type seems to have been first suggested by Steger [16] using hyperbolic equations. Hyperbolic grid generation equations were used by Matsuno [17] to extrude a prismatic volume mesh from a triangular surface mesh with only limited success. A mixed hexahedral/prismatic, near-body grid generation algorithm was employed by Wey for generating Chimera grids [18]. In [18], an unstructured quadrilateral/triangle surface mesh was extruded into the domain after the surface normals used for extrusion were smoothed. In this effort, we have employed a modified form of Knupp's approach [19] for applying Winslow smoothing [20] to the extruded mesh of arbitrary topology.

Near-Body Mesh Generation

In the near-body, generalized mesh generation algorithm presented here, the mesh is extruded in layers starting from an initial surface mesh of arbitrary topology. Generation of the mesh within each layer is accomplished using a modification of the parabolic mesh generation strategies developed for structured meshes [3]-[5] and semistructured meshes [14][15] and can be described as a three-step process:

- A three-level, locally orthogonal, reference mesh is generated by extrusion of the initial surface mesh in the direction of the local surface normal following a prescribed distance distribution. The reference mesh consists of the initial data surface for the layer and two extruded surfaces.

- The intermediate level of the reference mesh is iteratively smoothed using the Poisson smoothing equations from structured grid generation. The third level of the mesh is adjusted after each iteration to reflect the changes in the surface normals of the second level as it is smoothed.

- The third level of the reference mesh is then discarded and the second level is used as the initial data surface for the next layer. The process is then repeated.

The resulting smoothed mesh still exhibits many of the characteristics of the reference mesh. In this respect, it is fair to characterize the mesh as nearly orthogonal. The greatest deviation from orthogonality occurs where the smoother has done the most work. Typically, this occurs in regions where grid lines emanate from concave or convex regions of the initial surface.

Poisson Smoothing

One unique feature of this approach is the use of the Poisson grid generation algorithm to smooth the surfaces of the mesh as the extrusion proceeds. Laplacian smoothing is typically used to smooth the extruded surfaces [11]. Using the Poisson equation as the smoothing mechanism, while not as resistant to mesh folding as Winslow smoothing [19], allows the use of control functions to influence the spacing of points on the extruded surface. As shown in [14] and [15], the control functions play an important role in determining the quality of the extruded structured mesh.

In structured grid generation [1], a *global transformation* of the form

$$\xi = \xi(x, y, z), \eta = \eta(x, y, z), \zeta = \zeta(x, y, z) \qquad \text{(Eq. 1)}$$

is typically employed and the inverse transformation is assumed to exist. Assuming that ζ is the marching (extrusion) direction and that ζ lines are orthogonal to the extruded ζ=constant surface, the Poisson equation commonly used for structured grid generation becomes

$$\frac{g_{33}}{g^2}\left(g_{22}(\boldsymbol{r}_{\xi\xi} + \Phi \boldsymbol{r}_{\xi}) - 2g_{12}\boldsymbol{r}_{\xi\eta} + g_{11}(\boldsymbol{r}_{\eta\eta} + \Psi \boldsymbol{r}_{\eta})\right) + \frac{g_{11}g_{22} - g_{12}^2}{g^2}(\boldsymbol{r}_{\zeta\zeta} + \Theta \boldsymbol{r}_{\zeta}) = 0 \qquad \text{(Eq. 2)}$$

where $\boldsymbol{r}$(x,y,z) is the position vector, Φ, Ψ, and Θ are the control functions for the ξ, η, and ζ directions respectively, and the metrics g_{11}, g_{12}, g_{22}, g_{33}, and g are defined in the usual manner [1]. Note that the two-dimensional grid generation equations appear in parentheses in the first line of Eq. 2. In a structured grid generation algorithm, the partial derivatives appearing in Eq. 2 are approximated using standard second-order central differences. Clearly, an alternative approach is necessary if the Poisson equation is to be used for smoothing meshes of arbitrary topology.

Knupp [19] describes an approach for applying the original Winslow smoother [20], which was developed for a structured triangular mesh having six-valent nodes, to a two-dimensional mesh of arbitrary topology. Here we include the control functions so that mesh quality may be improved. As noted in [19], if a Laplace-type equation is to be used for smoothing an unstructured mesh, the notion of a *global coordinate transformation* must be abandoned. Using the notation of [19], a *local, discrete, uniform logical space* (ξ_m,η_m) is defined at each node

$$\xi_m = \cos\theta_m,\ \eta_m = \sin\theta_m \qquad \text{(Eq. 3)}$$

where $\theta_m = 2\pi m / M$, M is equal to the number of valent nodes, and m=0,1,…,M-1. Approximations for the partial derivatives f_ξ, f_η, $f_{\xi\xi}$, $f_{\eta\eta}$, and $f_{\xi\eta}$ for a generic function f are given in [19] for the necessary valent node combinations.

It should be noted that Knupp's approach cannot directly be extended to three dimensions for general unstructured meshes since the necessary logical space cannot be defined. However, since we are dealing with meshes generated by extrusion, the resulting underlying structure of the extruded mesh can be exploited to evaluate the derivative terms in the direction of extrusion, i.e., the ζ derivatives, using standard finite-difference approximations. The remaining derivative terms in Eq. 2 are approximated using Knupp's approach. Currently, a simple Jacobi iteration is used to solve the resulting system of equations.

Control Functions

One advantage of using the Poisson mesh generation equations is that many of the important results from elliptic grid generation, in particular those associated with the control functions, can be employed to generate high quality grids [14][15]. The form of the control functions used here does not include curvature effects [21] and is given by

$$\Phi = -\frac{1}{2}\frac{\partial}{\partial\xi}\ln\left(g_{11}\right),\ \Psi = -\frac{1}{2}\frac{\partial}{\partial\eta}\ln\left(g_{22}\right),\ \Theta = -\frac{1}{2}\frac{\partial}{\partial\zeta}\ln\left(g_{33}\right) \qquad \text{(Eq. 4)}$$

The strategy currently employed is to compute the control functions Φ and Ψ once using values from the initial surface mesh. The control function Θ is computed for each layer using the specified marching distance distribution. Additionally, all derivatives appearing in Eq. 4 must be evaluated using the local coordinate system at each point.

Additional Smoothing

On occasion, it has been found that, when using the parabolic marching scheme, adding dissipation to the Poisson smoothing equation (Eq. 2) is necessary, particularly when generating meshes in strongly nonconvex regions [14][15]. The approach employed here is described in detail in [22] and is similar in spirit to the approach taken by Chan and Steger [2].

Sample Meshes

To provide a demonstration of the efficacy of this approach, several preliminary three-dimensional meshes are included. These examples were chosen to illustrate the characteristics of the smoothing algorithm. Before extruding from the initial surface mesh, a check is performed to ensure that all faces are defined in a consistent clockwise direction so that the direction of extrusion is properly defined. In the examples shown below, the marching distance distribution is defined using a hyperbolic tangent stretching function. For purposes of improved visual clarity, the meshes included here were generated using a small number of points. The volume mesh is stored in the Cobalt_{60} mesh format [23]. Included are the cell connectivity information, the physical coordinates of each vertex, and a boundary flag. The EnSight software package [24] with the Cobalt_{60} reader was used to visualize the mesh.

The first sample mesh is for a simple bump. The surface mesh defining the bump was a structured, 31x21 quadrilateral grid. Figures 1 and 2 show two different views of the mesh. The smoothing algorithm was applied 10 times in each layer. The mesh appears smooth. As shown in Figure 2, the mesh exhibits clustering of lines emanating from concave regions of the surface and divergence of lines emanating from convex regions. Both behaviors are characteristic of meshes generated using marching methods. In general, the mesh retains its nearly orthogonal nature.

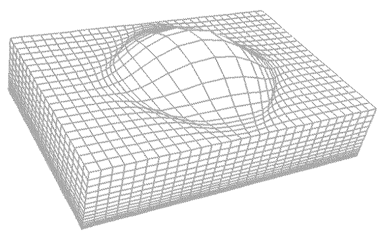

Figure 1. Isometric view of simple bump mesh

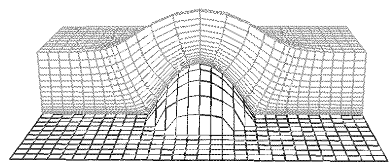

Figure 2. Cutaway view of simple bump mesh

Figures 3, 4, and 5 show the mesh generated around a body of revolution based on notional Apollo command and service modules. The initial surface mesh was defined using 1580 quadrilaterals and 20 triangular faces at the nose. The direction of march at the aft end of the nozzle is constrained to lie in the plane perpendicular to the axis of revolution. Figure 3 shows the symmetry plane of the extruded mesh. Figure 4 shows a detailed view of the mesh in the nonconvex region near the nozzle. As seen in the figure, the grid lines are orthogonal except at those regions where mesh lines converge, i.e., where the Poisson smoother does the most work. Figure 5 shows the detailed view of the nose. As can be seen from the figure, the point at the nose is a 20-valent node. Although the mesh lines in this region diverge because of the convexity of the corner, the distribution near the nose is observed to be very smooth.

The final example is for a finned-missile configuration. The surface of the missile was defined using 7934 triangles and the mesh was extruded nine layers. Ten smoothing iterations were performed in each layer. Figure 6 shows a cutaway view of the symmetry plane of the mesh. At the aft end of the missile, the mesh was constrained to lie in an x=constant plane. No smoothing was applied in this plane. Figure 7 shows an isometric view of the surface of the outer layer of the mesh. As can be seen from the figure, the mesh appears qualitatively smooth. Figure 8 shows a view of the outer layer near the fin root.

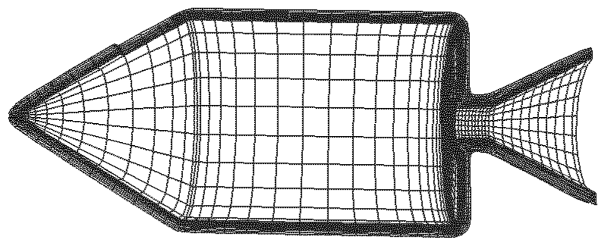

Figure 3. Symmetry plane of mesh generated around notional Apollo spacecraft

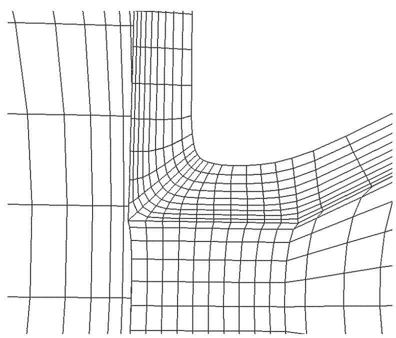

Figure 4. Detail of nonconvex region near nozzle

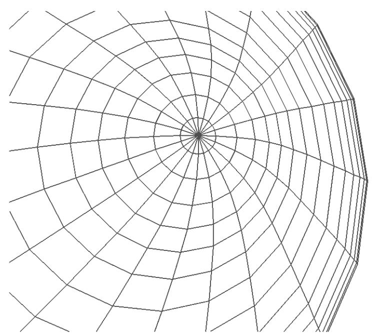

Figure 5. Detail of region near nose

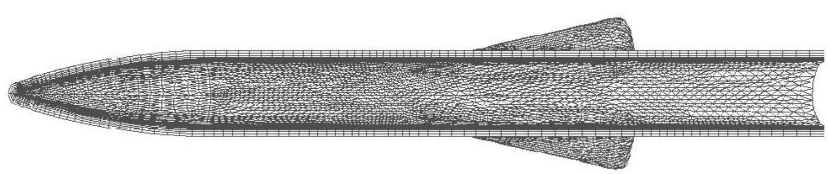

Figure 6. Symmetry plane of mesh generated around finned-missile

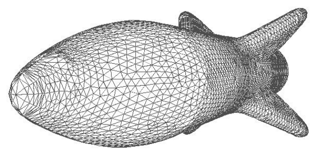

Figure 7. Isometric view of mesh outer surface

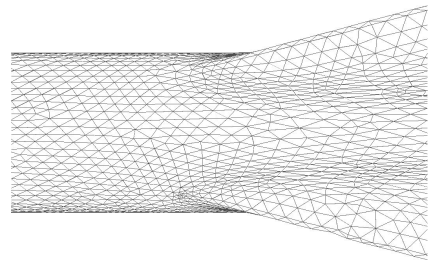

Figure 8. Detail of mesh outer surface near fin root

Here the effects of the marching scheme are observed in the high aspect ratios of some of the triangles near the fin/body juncture. This effect is analogous to the clustering of grid lines emanating from concave regions of the initial surface that was observed in Figures 1 and 2 for a structured mesh. It should be noted that some of the observed effect is due to the observer's position looking down on the fin.

Summary

A novel algorithm to extrude smooth, near-body volume meshes from surface meshes of arbitrary topology has been presented. The meshes are classified as generalized meshes because multiple element topologies may be present within the same mesh. The algorithm utilizes a three-step, parabolic scheme based on the Poisson equation used in structured grid generation to extrude the volume mesh. Structure of the mesh in the direction of extrusion was exploited so that a surface-smoothing algorithm for unstructured meshes could be employed. The technique was demonstrated for surface meshes consisting of a structured quadrilateral grid, a quadrilateral-dominant, mixed surface mesh, and an unstructured surface mesh. Presently, the algorithm is not yet mature. Further research needs to be performed to determine the best form of the control functions for the mixed topology meshes. Additionally, improvements in algorithm efficiency are needed.

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